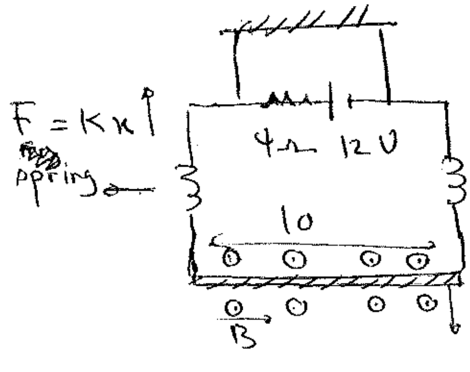
HW # 8

5.3



$\longrightarrow I = \frac{12}{4} = 3A$

$\longrightarrow m = 20g = 20\times10^{-3}$ Kg

$d = 10cm = 0.1m$

$F_{mg} = mg$

$dF = I\vec{dl} \times \vec{B}$

$\longrightarrow F_{spring} = Kx + Kx,$

$F_g = mg$

$\longrightarrow K = \frac{mg}{2x} = \frac{(20\times10^{-3})(9.81)}{2(2\times10^{-3})}$

$F_{\vec{B}} = IlB = 2Kx' = 2K(5\times10^{-3})$

$\longrightarrow IlB = 2Kx'$

$B = \frac{2mg}{2x} \cdot \frac{x'}{Il} = \frac{mg}{Il} \cdot \left(\frac{x'}{x}\right)$

$\longrightarrow B = \frac{(20\times10^{-3})(9.81)(5\times10^{-3})}{3\times(0.1)(2\times10^{-3})} = 1.635$ Tesla

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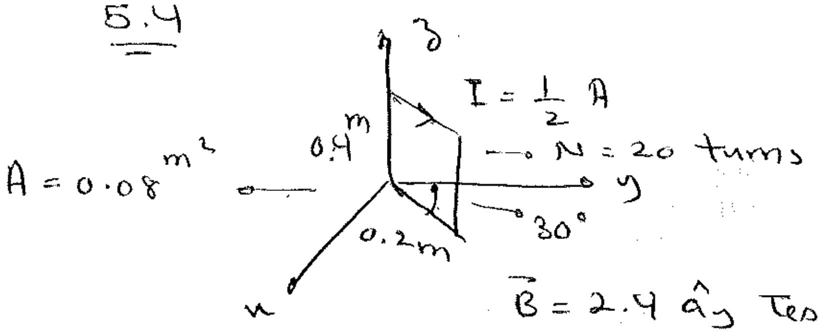
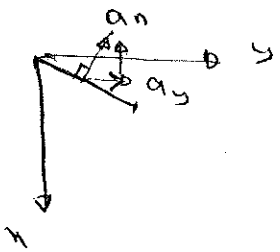
5.4



$\vec{B} = 2.4\,\hat{a}_y$ Tesla

x-y plane,



$\longrightarrow \hat{a}_n = -\frac{\sqrt{3}}{2}\hat{a}_x + \frac{1}{2}\hat{a}_y$

$\vec T = \vec m \times \vec B$

$\vec m = NIA\,\hat a_n \;\longrightarrow\; \vec T = NIAB\left(\frac{\sqrt3}{2}\hat a_x + \frac{1}{2}\hat a_y\right)\times(\hat a_y)$

a

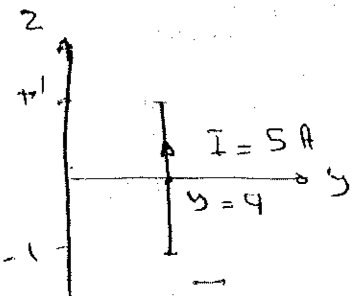
$\hat a_z$

$= (20)\left(\frac{1}{2}\right)(0.08)(2.4)\left(-\frac{\sqrt3}{2}\right)\hat a_z$

$= -1.663\,\hat a_z$

$\longrightarrow$ clockwise torque

5.5



$\vec B = 0.2\cos\varphi\,\hat a_r$

$\vec B = B_r\,\hat a_r$

$d\vec F = I\,d\vec l \times \vec B$

$d\vec l = dz\,\hat a_z$

$\longrightarrow d\vec F = I\int_{-1}^{1} dz\,\hat a_z \times B_r\,\hat a_r = I B_r z\,\hat a_\varphi$

$\longrightarrow \vec F = (5)(0.2\cos\varphi)(z)\,\hat a_\varphi$

$= 2\cos\varphi\,\hat a_\varphi$

at $\varphi = \pi/2 \longrightarrow F = 0$

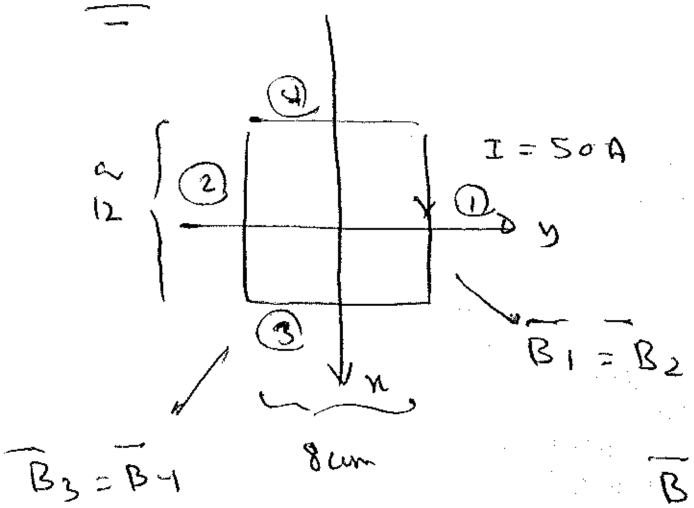
$W = \int \vec F \cdot d\vec l$

$dl = -r\,d\varphi\,\hat a_\varphi \big|_{r=4} \longrightarrow W = \int_0^{2\pi} 2\cos\varphi\,\hat a_\varphi \cdot (-4\,d\varphi\,\hat a_\varphi)$

$= -8\sin\varphi\Big|_0^{2\pi} = 0$

$\vec F = 2\cos\varphi\, a_\varphi \longrightarrow F_{max} = 2$ at $\varphi = 0 \longrightarrow \vec F = 2\,\hat a_\varphi$

5.7



$$B = \frac{\mu_0 I l}{2\pi r\sqrt{4r^2 + l^2}}\,\hat{a}_\varphi$$

For ① $\longrightarrow$ $\overline{B}_1 = \dfrac{-\mu_0(50)(12a)}{2\pi(4a)\sqrt{4(4a)^2 + (12a)^2}}\,\hat{a}_z$

$$\longrightarrow \overline{B}_1 = \frac{300000}{\sqrt{208}} \times 10^{-7}$$

$$\overline{B}_3 = \frac{-\mu_0(50)(8a)}{2\pi(6a)\sqrt{4(6a)^2 + (8a)^2}}\,\hat{a}_z$$

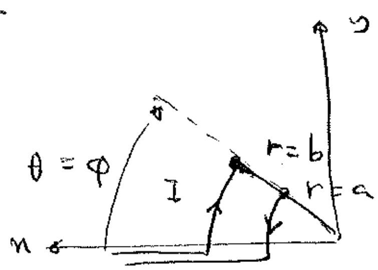
$$\longrightarrow \overline{B}_3 = \frac{40000}{3\sqrt{208}} \times 10^{-7}$$

$$\longrightarrow \overline{B}_{total} = 2\overline{B}_1 + 2\overline{B}_3$$

$$= -6.01 \times 10^{-4}\,\hat{a}_z$$

$$= -0.6\,\hat{a}_z \text{ mtesla}$$

5.9



$$d\overline{H} = \frac{I\,d\overline{l} \times \overline{R}}{4\pi R^3}$$

$\overline{dl}_b = b\,d\varphi\,\hat{a}_\varphi \longrightarrow \overline{R}_b = -b\,\hat{a}_r$

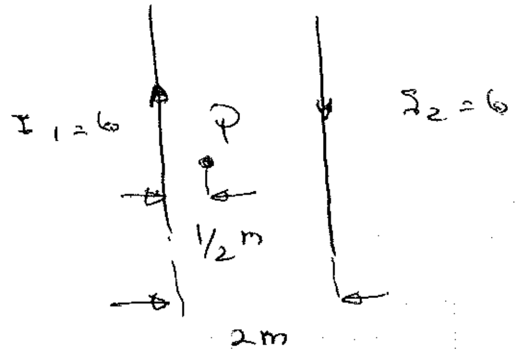
$\overline{dl}_a = -b\,d\varphi\,\hat{a}_\varphi \longrightarrow \overline{R}_a = -a\,\hat{a}_r$

$$\longrightarrow \overline{H} = \overline{H}_b + \overline{H}_a$$

$$= \frac{I b^2 \varphi}{4\pi b^3}\,\hat{a}_z - \frac{I a^2 \varphi}{4\pi a^3}\,\hat{a}_z$$

$$= \frac{I\varphi}{4\pi}\left(\frac{1}{b} - \frac{1}{a}\right)\hat{a}_z$$

5.12



$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{H} = \frac{I}{2\pi r} a_\varphi$$

$$\vec{B}_{1p} = \frac{6\mu_0}{2\pi(\frac{1}{2})} \hat{a}_{in}$$ into the page

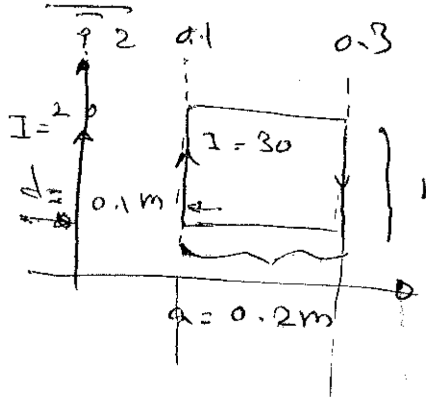
$$\vec{B}_{2p} = \frac{6\mu_0}{2\pi(\frac{3}{2})} \hat{a}_{in}$$

$$\vec{B} = \vec{B}_{1p} + \vec{B}_{2p}$$

$$= \frac{3\mu_0}{\pi}\left(\frac{1}{1/2} + \frac{1}{3/2}\right)$$

$$= \frac{8\mu_0}{\pi}\hat{a}_{in} = 32 \times 10^{-7} \hat{a}_{in} = 3.2\, \hat{a}_{in} \ \mu\text{testa}$$

5.15



$$\vec{B}_{0.1} = \frac{\mu_0 I_1}{2\pi d} \hat{a}_\varphi$$

$$\vec{B}_{0.3} = \frac{\mu_0 I_1}{2\pi (d+a)} \hat{a}_\varphi$$

$$\vec{F} = I \vec{l} \times \vec{B}$$

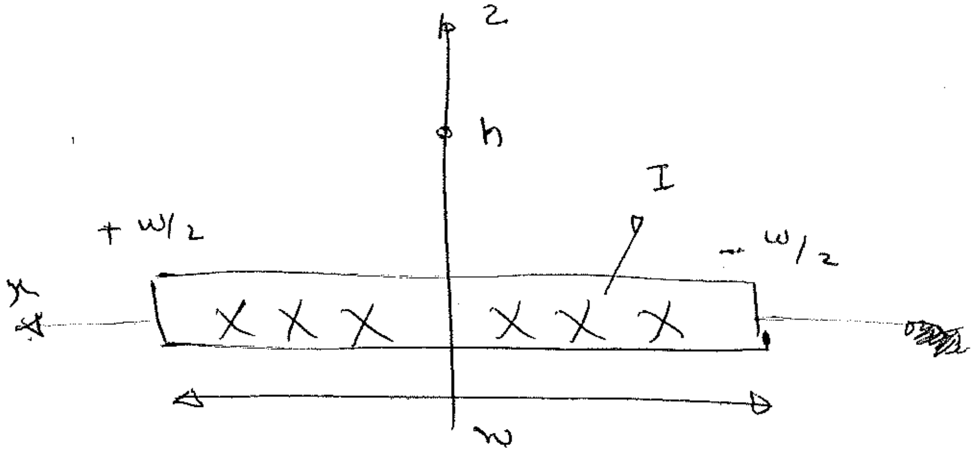
$$\vec{F}_{0.1} = I_2 b \hat{a}_z \times \frac{\mu_0 I_1}{2\pi d} \hat{a}_\varphi = \frac{-\mu_0 I_1 I_2 b}{2\pi d} \hat{a}_r$$

$$\vec{F}_{0.3} = -I_2 b \hat{a}_z \times \frac{\mu_0 I_1}{2\pi (a+d)} \hat{a}_\varphi = + \frac{\mu_0 I I_2 b}{2\pi (a+d)} \hat{a}_r$$

$$\vec{F} = \vec{F}_{0.1} + \vec{F}_{0.3} = \frac{-\mu_0 I_1 I_2 b}{2\pi}\left(\frac{1}{d} + \frac{1}{a+d}\right)\hat{a}_r$$

$$= \frac{-2(20)(30)(1/5)(1/2)}{1/10\,(3/10)} \times 10^{-7} \hat{a}_r = -0.4\hat{a}_r \ \text{mN}$$

5.17



$$\overline{H} = \frac{I}{2\pi r}\hat{a}_\varphi \quad \text{and} \quad J_s\,dx = \frac{I}{w}\,dx$$

considering the symmetry $\longrightarrow$ $\hat{a}_\varphi = -\frac{h}{\sqrt{x^2+h^2}}\hat{a}_x - \frac{x}{\sqrt{x^2+h^2}}\hat{a}_z$

$$\longrightarrow \overline{dH} = \frac{-\frac{I}{w}\,dx}{2\pi\sqrt{x^2+h^2}}\left\{\frac{h}{\sqrt{x^2+h^2}}\hat{a}_x + \frac{x}{\sqrt{x^2+h^2}}\hat{a}_z\right\}$$

again by symmetry $\longrightarrow$ $\hat{a}_z$ components will cancel

$$\longrightarrow H_x = -\frac{Ih}{2\pi w}\int_{-w/2}^{w/2}\frac{dx}{x^2+h^2} = \frac{-Ih}{2\pi w}\times\frac{1}{h}\,tg^{-1}\frac{x}{h}\Big|_{-w/2}^{w/2}$$

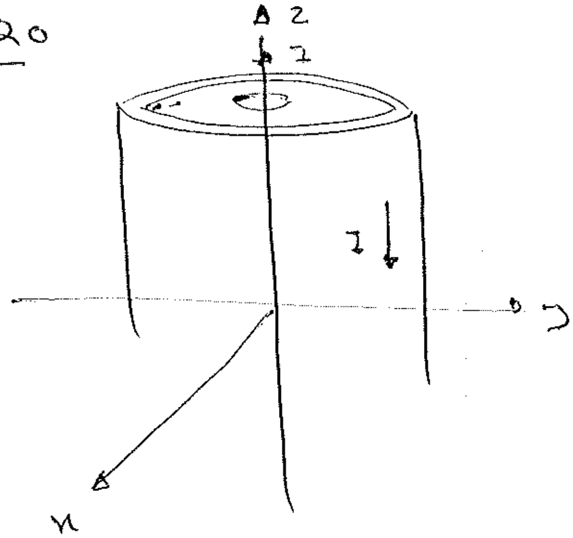
$$= -\frac{I}{\pi w}\,tg\,\frac{w}{2h}\,\hat{a}_x \quad \text{at } (0,0,h)$$

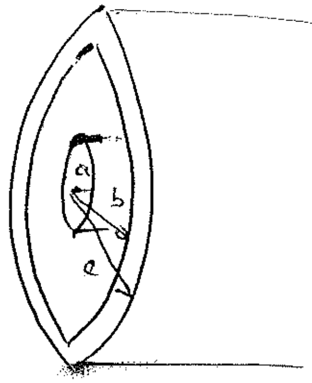
(b) $\overline{dF} = I\,\overline{dl}\times\overline{B} = I\,dy\,\underbrace{\hat{a}_y \times \frac{-\mu_0 I}{\pi w}\,tg\left(\frac{w}{2h}\right)\hat{a}_x}_{\hat{a}_z}$

$$= \frac{\mu_0 I^2}{\pi w}\,tg\left(\frac{w}{2h}\right)dy\,\hat{a}_z$$

$$\longrightarrow dF = \int_{-l/2}^{l/2}\overline{dF} = \frac{\mu_0 I^2}{\pi w}\,tg\left(w/2h\right)l\,\hat{a}_z \longrightarrow \overline{F}/l = F' = \frac{\mu_0 I^2}{\pi w}\,tg\left(\frac{w}{2h}\right)\hat{a}_z$$

5.20





$r < a$: $I = \oint \vec{H}\cdot d\vec{l} = \int \vec{J}\cdot d\vec{s}$

$\vec{J} = \frac{I}{\pi a^2}\hat{a}_z$, $\vec{H} = \frac{I'}{2\pi r}\hat{a}_\varphi$, $I' = \int_0^{2\pi}\int_0^{r} \frac{I}{\pi a^2}\, r\,dr\,d\varphi$

$$I' = \frac{I}{\pi a^2}\cdot\frac{2\pi r^2}{2} = I\left(\frac{r^2}{a^2}\right)$$

$$\longrightarrow \vec{H} = \frac{I\left(\frac{r}{a}\right)^2}{2\pi r}\, a_\varphi$$

$$\longrightarrow \vec{H}_1 = \frac{I}{2\pi a^2}\, r\, \hat{a}_\varphi \quad 0 \le r \le a$$

$a < r < b$

$$\vec{H}_2 = \frac{I}{2\pi r}\hat{a}_\varphi$$

$b < r < c$: $J = -\frac{I}{\pi(c^2-b^2)}\hat{a}_z \longrightarrow I' = I - \int_0^{2\pi}\int_b^{r} \frac{I}{\pi(c^2-b^2)}\, r\,dr\,d\varphi$

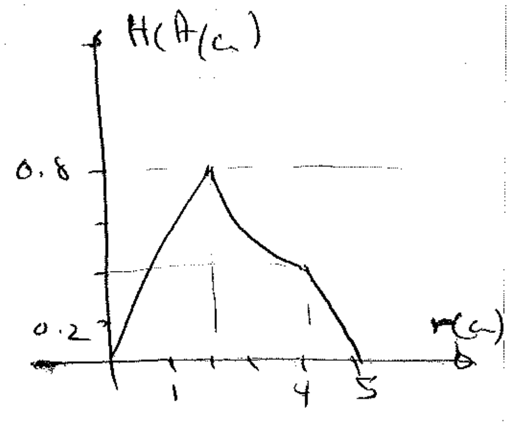
$$I' = I - I\left(\frac{r^2-b^2}{c^2-b^2}\right) = I\left(\frac{c^2-r^2}{c^2-b^2}\right)$$

$$\longrightarrow \vec{H}_3 = \frac{I\left(\frac{c^2-r^2}{c^2-b^2}\right)}{2\pi r}\, a_\varphi$$

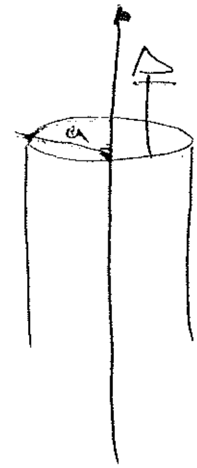
(b)

$I = 10$ A

$\begin{cases} a = 2 \\ b = 4 \\ c = 5 \end{cases}$ a $\longrightarrow H_{max} = \frac{I}{2\pi a} = \frac{1000}{4\pi} = 0.79$ A/c



5.21



$\vec{J} = J_0/r \, \hat{a}_z$

$r \le a$

$$I = \oint \bar{H}\cdot d\bar{l} = \int_s \bar{J}\cdot d\bar{s}$$

$$H_\varphi \int r\,d\varphi = \int_0^{2\pi}\int_0^{r} J_0/\cancel{r}\,\cancel{r}\,dr\,d\varphi$$

$$\longrightarrow H_\varphi \cdot r \cdot 2\pi = 2\pi J_0 \cdot r \longrightarrow H_\varphi = J_0$$

$$\longrightarrow \vec{H} = J_0\,\hat{a}_\varphi \qquad 0 \le r \le a$$

$a < r$

$$I = 2\pi J_0 a$$

$$\longrightarrow H_\varphi (r \times 2\pi) = 2\pi J_0 a$$

$$\longrightarrow H_\varphi = \frac{J_0 a}{r}$$

$$\longrightarrow \vec{H} = J_0 \frac{a}{r}\hat{a}_\varphi \qquad a \le r$$

5.23

$$\bar{H} = 4/r\,[1-(3+3r)e^{-3r}]\,\hat{a}_\varphi$$

$$\bar{J} = \bar{\nabla}\times\bar{H}$$

$$\begin{cases} H_r = 0 \\ H_z = 0 \end{cases} \longrightarrow \vec{\nabla}\times\vec{H} = \cancel{-\frac{\partial H_\varphi}{\partial z}}\,\hat{a}_r + \frac{1}{r}\frac{\partial (rH_\varphi)}{\partial r}\,\hat{a}_z$$

$$\longrightarrow \vec{\nabla}\times\vec{H} = \frac{1}{r}\frac{d(rH_\varphi)}{dr}\,\hat{a}_z = J$$

$$\longrightarrow \bar{J} = \frac{1}{r}\frac{d}{dr}\left[4\,[1-(1+3r)e^{-3r}]\right]\hat{a}_z$$

$$= 36e^{-3r}\,\hat{a}_z \quad \text{Amp.}/m^2$$